Rational approximants for vector of analytic functions with branch points and extremal problems for vector potentials.

Alexander I. Aptekarev,

Keldysh Institute of Applied Mathematics
Russian Academy of Sciences
Moscow, Russia

Given a vector of power series expansions at infinity point which allows analytic continuation along any path of complex plane non-intersecting with a finite set of branch points. For this set of functions Hermite-Pade rational approximants are considered. The following extremal problem is proposed for finding the limiting distributions of the poles and the extra-interpolation points for these approximants. There is a vector of measures supported on the fixed system of curves and arcs. Extremal problem of minimization of logarithmic potential energy for the system of measures with a special interaction matrix is considered. Then by variation of the system of curves and arcs we find locations for the curves where the local maxima of the energy functional is achieved. As a result, an extremal system of curves and arcs arises. These extremal contours and the extremal measures on them give the limiting distributions of the poles and the extra-interpolation points of Hermite-Pade approximants. The application of the Matrix Riemann-Hilbert problem method for the asymptotic analysis of these approximants uses the extremal system of contours for finding an appropriate deformation of the boundaries for the solution of BVP for matrix analytic functions.