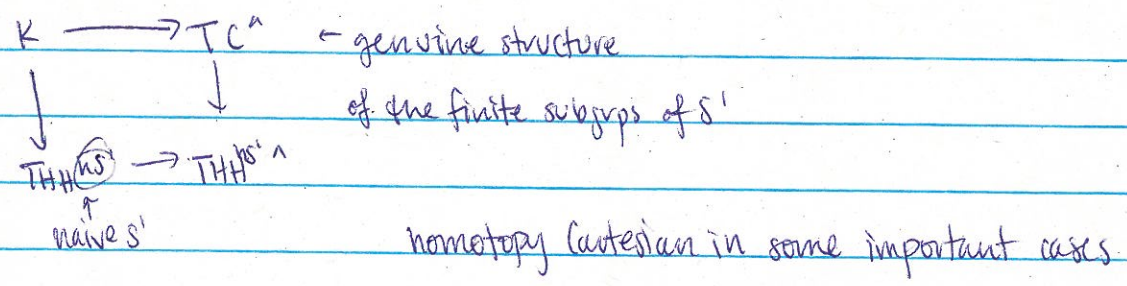
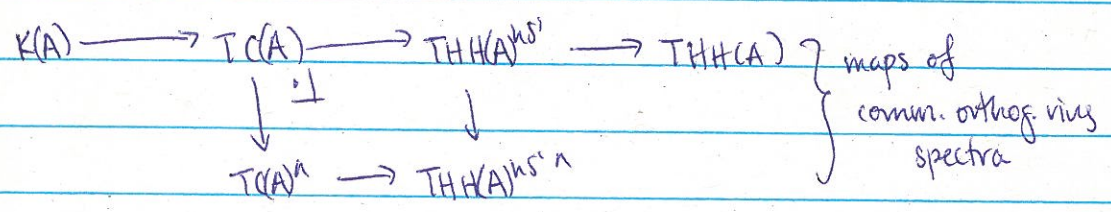


Bjorn Dundas: 2/13/2012

- 1.) "Why does the standard smash product fail to give a model for THH?"
- 2.) "What part of  $S^1$  is important in the eyes of K-theory?"
- 3.) "What are the slices of THH?"



A commutative orthogonal ring spectrum (connective)



Rognes' redshift conjecture:

What are the conditions on  $A$ , so that

$$\text{telecom } K(A) = \text{telecom } A + 1 \quad (\text{telecom} = \text{telescopic complexity})$$

Ex:  $K(\mathbb{F}_p) \simeq H\mathbb{Z}_p \simeq \mathbb{Z}_p$  telecom 0

$K(K(\mathbb{F}_p)_p) \simeq J_p \times B\mathbb{Z}_p \times SU_p \simeq \mathbb{Z}_p$  telecom 1

$K(K(K(\mathbb{F}_p)_p)_p) \simeq \mathbb{Z}_p$  Avsoni theorem telecom 2

} come from TC  
 $\pi_0 A = \mathbb{Z}_p$   
 $KA_p \rightarrow TCA_p \rightarrow \Sigma^{-1} H\mathbb{Z}_p$   
 Hesselholt-Madsen

TC ... TC bad functor (mixes limits + colimits  $\Rightarrow$ ) ← only for detecting!

pull out all the limits:  $((A \wedge A)^{\wedge 2} \wedge (A \wedge A)^{\wedge 2})^{\wedge 2}$  vs.  $(A \wedge A \wedge A \wedge A)^{\wedge 2 \times 2}$

$$THH(A) = A \otimes S^1$$

$$THH \cdots THH(A) = (A \otimes S^1) \otimes \cdots \otimes S^1 = A \otimes T^n$$

$$K \cdots K(A) \longrightarrow TC \cdots TC(A)$$

$$\downarrow \quad \downarrow$$

$$? \longrightarrow GL_n(\mathbb{Q}_p) \hookrightarrow T^n C(A, P)$$

covering homotopy finite subgrps of  $T^n$   
 $P: T^n \rightarrow T^n$

Constructing  $T^n C(A, P)$ :

$$\begin{array}{ccc} X & \longrightarrow & \Lambda_X A \\ \uparrow & & \\ \text{space} & & \end{array} \quad \begin{array}{l} G \text{ finite, } X \text{ free } G\text{-space} \\ \text{Bö-type construction.} \end{array}$$

$[\Lambda_X A]^G$  built from  $\{(\Lambda_{X/H} A)_{h \in W_G H}\}$  conj. classes  $H \leq G$ .

How are these assembled?

"ghost map"  $[\Lambda_X A]^G \longrightarrow \prod_{\substack{H \leq G \\ \text{conj}}} \Lambda_{X/H} A$

$$\pi_0 [\Lambda_X A]^G = W_G \pi_0 A \quad (\text{Burnside-Witt})$$

$X \text{ connected}$

Often compared via the Hasse square:

$$\begin{array}{ccc} [\Lambda_X A]^G & \longrightarrow & \Lambda_X A^{g^G} \\ \downarrow \mu & & \end{array}$$

$$[\Lambda \Lambda_X A]^{g^G} \longrightarrow \Lambda_X A^{+G}$$

1.) Ex:  $A = \mathbb{Z}_p, X = S^1$  BM & T solidus  $\Gamma$  eq. in pos. degrees

2.) SW & R:  $X$  finite,  $A$  finite type  $\Gamma$  eq.

3.)  $(\Lambda_{\mathbb{Z}^2} H\mathbb{F}_2)^{h\mathbb{T}^2}$  detects  $v_1$  Pognes

$(\Lambda_{\mathbb{T}^2} H\mathbb{Z}_p)^{h\mathbb{T}^2}$  "  $v_2$  Veen

→ maybe there is a next step.

Categorical smash ~~product~~ powers:  $[Sto|Z]$

$G$  compact Lie

$$\text{orth. } G\text{-spectra} \xrightarrow{\text{ev}_V} \mathcal{O}_V \times G\text{-space}$$

$$\begin{array}{ccc} X & \longrightarrow & X_V \\ \downarrow & & \downarrow \end{array} \quad \begin{array}{l} \mathcal{O}_V \times G \text{ space, since } G \rightarrow \mathcal{O}_V \end{array}$$

want to lift model structure from space level & stabilize

→ need to take care about what equivalences to use on the spaces.

choose a family of subgroups of  $G, H$ .

$$\begin{array}{ccc} \mathcal{F} \equiv \{P \equiv \mathcal{O}_V \times G\} & & \mathcal{O}_V \times H \\ \downarrow & \text{inj} \searrow & \downarrow \\ \mathcal{H} \equiv \{\text{subgrp of } G\} & & H \end{array} \quad \begin{array}{l} \text{sect.} \uparrow \end{array}$$

$O_V \times G$ -spaces: model structure

Weak eq. =  $\mathcal{F}$ -w.e.

equiv. cof. with all orbit types.

cof = genuine cof  $\leftarrow$  mixed model structure. (in sense of Shipley)

fib = RLP

Gives level structure for orthog.  $G$ -spectra:

(fib doesn't depend on universe yet.)

w.e/fib = w.e/fib in each level

cof = LLP.

compactly gen. model cat.

Stabilize: w.e.  $\pi_*$ -isomorphism:  $\pi_*^H$ -isos  $H \in \mathcal{H}$

cof = level cof

fib = RLP.

Theorem (Stolz)

positive (wrt fixed points?)

Commutative orthogonal  $G$ -ring spectra  $\xrightleftharpoons[\text{forget}]{\text{free}}$  orthog.  $G$ -spectra

w.e/fib = w.e/fib in orthog.  $G$ -spectra

- get compactly generated model structure.
- forget preserves cofibration.

Filtration of smash powers:

$G$  finite.  $S$  finite free  $G$ -set

$M$  cellular orthog. spectrum.

$M^{nS}$  cellular  $G$ -orthog. spectrum.

$G \wedge_{H_*} G_V \left( (S^{n-1} \cong D^1) \times O_V \right)^{\square H} \leftarrow$  give the type of the cells.  
 $H \leq G \quad V \quad P \leq O_V$

so:  $M^{nS/H} \cong (M^{nS})^{\square H}$

Enough functoriality to push this up to tori

$X$  naively cof.  $T^n$ -space  $H \leq T^n$  finite.

$A$  cof. comm. orthog. ring spectrum. Then: nat. isom of  $T^n/H$ -comm. orth. ring sp

$A \otimes_X \mathbb{Z}/H \cong (A \otimes X)^{\square H}$  upshot: Bökstedt-type construction agrees w/ categorical.

isotropy separation sequence:

$$(\Lambda_x A)_{\mathbb{N}^n} \rightarrow [\Lambda_x A]_{\mathbb{R}}^n \xrightarrow[\substack{\text{holim} \\ \mathbb{N} \geq \mathbb{H} \neq 0}]{\text{holim}} [\Lambda_{x/\mathbb{H}} A]_{\mathbb{H}}^{n/\mathbb{H}}$$

$\nearrow$   
 $[A \otimes X]_{\mathbb{H}}^n$   
 theorem of pres. equiv.  $A \otimes X / \mathbb{H} \cong (A \otimes X)^{\mathbb{H}}$  + induction