SLANTED MATRICES, BANACH FRAMES, WIENER'S LEMMAS AND INVERSE PROBLEMS

The class Σ_α^w of slanted matrices with ω-*summable* α-slants consists of matrices $\mathbb{A}$ such that $\|\mathbb{A}\|_{\Sigma_\alpha^\omega} = K \sum_{j\in\mathbb{Z}^d} \left\|A_j^\alpha\right\|_{\sup} \omega_j < \infty$, where A_j^α is the j^{th} slanted line of $\mathbb{A}$ with slope α, e.g., A_0^1 is the diagonal of $\mathbb{A}$. If for some $p \in [1,\infty]$, $\|\mathbb{A}x\|_p \geq k_p \|x\|_p$ for all $x \in \ell^p$, where $k_p > 0$, then we prove that $\|\mathbb{A}x\|_q \geq k_q \|x\|_q$ for all $x \in \ell^q$ for all $q \in [1,\infty]$, as long as the weight function ω has reasonable decay. This result has many applications in Banach frame theory, generalizations of Wiener's Lemma, and to sampling theory.