Math 196 - Quiz 5, October 2, 2008 Name:______________________________

Problem 1. In each of the following cases determine whether or not the subset W is a subspace of the vector space V.

(1) W is the set of all vectors in $V = \mathbb{R}^3$ such that $x_1 + x_2 = 0$.

(2) W is the set of all vectors in $V = \mathbb{R}^3$ such that $x_1 + x_2 = 1$.

(3) W is the set of all vectors in $V = \mathbb{R}^3$ such that $x_1 x_2 = 0$.

(4) $W = \{0\} \subset V = \mathbb{R}^4$.

(5) W is the set of all vectors of the form $s\begin{pmatrix} 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ where $s, t \in \mathbb{R}$ and $V = \mathbb{R}^2$.

(6) W is the set of all vectors of the form $s\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or of the form $t\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ where $s, t \in \mathbb{R}$ and $V = \mathbb{R}^2$.

(7) W is the set of all vectors v in $V = \mathbb{R}^2$ such that $\begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix} v = 0$.

(8) W is the set of all matrices A in $V = M_{2\times 2}(\mathbb{R})$ such that $A\begin{pmatrix} 1 \\ 4 \end{pmatrix} = 0$.

(9) W is the set of all polynomials p such that $p(2) = 0$, where V is the vector space of all polynomials.

(10) W is the set of all polynomials p such that $p(0) = 2$, where V is the vector space of all polynomials.

Solution 1.

1. W is a solution set to a homogeneous system of linear equations and hence it is indeed a subspace.

2. W does not contain the 0 vector and is therefore not a subspace of V.

3. $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ are both in W but their sum $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ is not. Hence W is not a subspace.

4. W contains 0 by definition and is closed under addition and scalar multiplication because $0 + 0 = 0$ and $a0 = 0$, $\forall a \in \mathbb{R}$. Hence W is a subspace.

5. This is more familiar as the span of $\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\}$ and is indeed a subspace.

6. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ are both in W but their sum $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is not. Hence W is not a subspace.

7. W is just the set of solutions to the homogeneous system of linear equations given by

$$-x_1 + 2x_2 = 0$$

$$3x_1 + 2x_2 = 0.$$

Hence W is a subspace as in question 1.

8. $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

If $A, B \in W$ then $(A+B)\begin{pmatrix} 1 \\ 4 \end{pmatrix} = A\begin{pmatrix} 1 \\ 4 \end{pmatrix} + B\begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Hence $A+B \in W$.

If $A \in W$ and $a \in \mathbb{R}$ then $(aA)\begin{pmatrix} 1 \\ 4 \end{pmatrix} = a(A\begin{pmatrix} 1 \\ 4 \end{pmatrix}) = a\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Hence $aA \in W$.

Thus W contains the zero vector and is closed under addition and scalar multiplication and hence is a subspace.

9. If $p = 0$ then $p(2) = 0$ so $0 \in W$. If $p, q \in W$ then $(p+q)(2) = p(2) + q(2) = 0 + 0 = 0$ so $p + q \in W$. If $p \in W$ and $a \in \mathbb{R}$ then $(ap)(2) = a(p(2)) = a(0) = 0$ so $ap \in W$. Thus W is a subspace.

10. W does not contain the 0 polynomial and so is not a subspace.