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Hughes, Bruce [Hughes, C. Bruce]; Prassidis, Stratos

Control and relaxation over the circle. (English. English summary)

Mem. Amer. Math. Soc. **145** (2000), *no.* 691, *x*+96 *pp.*

This well-written research monograph obtains geometric analogues of the Bass-Heller-Swan fundamental theorem of algebraic K-theory for a ring R:

$$K_1(R[t,t^{-1}]) = K_1(R) \oplus K_0(R) \oplus \widetilde{\mathrm{Nil}}(R) \oplus \widetilde{\mathrm{Nil}}(R).$$

The Whitehead space $\mathcal{W}(X)$ of a finite CW-complex X is a space of homotopy equivalences to X from other finite CW-complexes. The main theorem is a homotopy equivalence:

$$\mathcal{W}(X \times S^1) \simeq \mathcal{W}(X) \times \Omega^{-1}\mathcal{W}(X) \times \widetilde{\mathcal{N}}(X) \times \widetilde{\mathcal{N}}(X),$$

involving a geometrically defined Nil space based on earlier work of S. Prassidis [K-Theory 5 (1991/92), no. 5, 395–448; MR 93e:57062]. There is also such a theorem for the controlled Whitehead space $\mathcal{W}(X \times S^1 \to S^1)$ of a compact Hilbert cube manifold X:

$$\mathcal{W}(X \times S^1 \to S^1) \simeq \mathcal{W}(X) \times \Omega^{-1}\mathcal{W}(X),$$

as well as for pseudoisotopy spaces. The techniques of proof involve ingenious geometric analogues of the algebraic proof of the original Bass-Heller-Swan result, making much use of controlled topology and manifold approximation fibrations. *A. A. Ranicki* (4-EDIN-MS)